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Relational Information Space for Dynamic Systems

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RELATIONAL INFORMATION SPACE FOR DYNAMICAL SYSTEMS

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ABSTRACT

Cooperative decentralized control of autonomous vehicles continues to be an important research subject for many military and civilian applications. Vehicles can communicate with each other and exchange information about their relative positions, target, environment, and use that data to develop decentralized but coordinated control policies. Communication is often represented by an information graph, and information exchange is modeled by a discrete-time dynamical system, known as the information loop. When the vehicles all agree on the information state it is said that they have reached an information "consensus," which is equivalent to conditional stability of the information loop. Often consensus control is slow to converge or easily destabilized. Being able to assess the quality of communication topologies (information graphs) is critical to determining the quality of a consensus solution.

In this work a topological manifold approach for representing complex networks of dynamical entities is shown to be an effective way to represent non-entropic measures of information as low dimensional embeddings in a high dimensional lifted space. A 2 dimensional embedding (stability margin - convergence rate) was developed that demonstrates an efficient frontier of graph topologies that dominate all others in their ability to reject disturbances and converge rapidly to a consensus.

TABLE OF CONTENTS

Introduction

The study of information dynamics is important for understanding complex networks of dynamical systems with information flows and shared or coordinated control policies. Information dynamics is essentially the study of information flows in a network when modeled as a discrete-time dynamical system. Control policies that shape the dynamics of the information flows in-turn shape the system dynamics of the physical entities within the network (e.g. formations, shared tasks). Vehicle communication is often represented by a communication or information graph, and because of possible delays in information processing and transmission, information exchange is modeled by a discrete-time dynamical system, known as the information loop. Ensuring desired stability and convergence properties of the information loop is critical to efficient performance of a cooperative system. When the vehicles all agree on the information state it is said that they have reached an information “consensus”

Optimizing the convergence rate of the information flow in consensus problems for various networks has long been an active research venue. On the other hand, stability of information flow in application to cooperative control of vehicle formation was comprehensively investigated, and a general method for deriving transfer functions for closed-loop multi-agent systems was recently suggested. However, often improvements of the convergence rate lead to degrading stability and vice-versa. Both stability and convergence are important characteristics of the information flow and should be analyzed together. This work develops an optimization framework for stability and convergence of the information flow in cooperative systems and also investigates the impact of the topology of the communication graph on the stability margin and convergence rate. The set of all graphs that that dominate all others in their ability to reject

disturbances and converge rapidly to a consensus was computed. This set is the efficient frontier or Pareto solution set. A high dimension “information space” may be constructed such that a point in this space corresponds to one possible graph and all possible graphs on n-nodes spans the space. The Pareto solution set is shown to be a 2 dimensional manifold on the information space.

The problem to be addressed is formulated as follows. Suppose there are n vehicles in a cooperative system. Let vector $x_i=(x_{i1},\ldots,x_{im})^T$ determine position of vehicle i in m-dimensional space, and let $X = (x_1,\ldots,x_n)$ be an $m\times n$ matrix describing position of the whole system. Communication in the system is represented by a directed communication graph G=(V,E) with V and E being the sets of vertices and edges, respectively, where node i represents vehicle i and the edge from i to j shows that vehicle j receives information from vehicle i (or “watches” vehicle i). Let N_i be the set of outgoing edges of vertex i with $|N_i|$ the cardinality of N_i. The normalized adjacency matrix G and the normalized Laplacian L of G are defined as follows:

$$G = \{g_{ij}\}_{i,j=1}^{n}, \qquad g_{ij} = \begin{cases} |N_i|^{-1}, & j \in N_i, \\ 0, & j \notin N_i \cup \{i\}, \end{cases}$$
$$L = I - G,$$

where I is the identity matrix.

Let $y_k = (y_{1k}, \ldots, y_{nk})$ be an error vector, in which component y_{ik} represents the error between an internal state measurement of vehicle i and a time-varying offset function $h_i(k)$ relative to an arbitrary reference at time mo ment $t = t_k$. For example, $y_{ik} = \|Cx_i(k) - h_i(k)\|$, where C is a matrix and $x_i(k)$ is the position of vehicle i at $t = t_k$. Also, let vector p_k denote information transmitted

to the vehicles at $t = t_k$. The information flow in the cooperative system can be represented by a discrete-time linear dynamical system (information loop)

$$\begin{aligned} q_{k+1} &= \sum_{j=0}^{M} a_j q_{k-j} + Gp_k + Ly_k, \quad k = 0,1,\ldots \\ p_k &= \sum_{j=0}^{M} b_j q_{k-j}, \quad k = 0,1,\ldots \end{aligned} \tag{1}$$

with $q_k = 0$ for $k = -M, \ldots, 0$; see [7].

We pose the following questions:

(i) Given a communication graph G, what is the best possible convergence rate for the information loop in (1)?

(ii) Given a communication graph G, what is the tradeoff between the convergence rate and stability margin as a function of the information control gains a_j ,b_j , $j = 0, \ldots ,M$?

(iii) Given a finite set of graphs on “n” vertices, what is the best topology of the communication graph G with respect to both the convergence rate and stability margin? Find an ordering of these graphs, from best to worst, that illustrates how some graphs can dominate others.

Convergence Rate.

Conditional convergence for a discrete LTI system is given as follows: G has exactly 1 eigenvalue λ_1=1 with eigenvector $\mathbf{1_N}$, all other eigenvalues are strictly within the unit disk. Conditional convergence is equivalent to "consensus" for an LTI information dynamical system.

Let $\lim_{k\to\infty} p_k = p^*$, $\lim_{k\to\infty} q_k = q^*$, $\lim_{k\to\infty} y_k = y^*$, $a = \sum_{j=0}^{M} a_j$, $b = \sum_{j=0}^{M} b_j$ and $c = (1-a)/b$

Then $q^* = aq^* + Gp^* + Ly^*$, $p^* = bq^*$

If c is an eigenvalue of G it must be 1 and unique with e.vector $\mathbf{1}_N$, can be shown

$p^* = y + \gamma \mathbf{1}_N$ with γ scalar multiplier that determines convergence

Define the block diagonal matrix

$$Q = \begin{bmatrix} N_0 & N_1 & \cdots & N_{M-1} & N_M \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \quad \text{where} \quad \begin{array}{l} N_j = a_j I + b_j G \\ \sum_{j=0}^{M} (a_j + b_j) = 1 \end{array} \tag{2}$$

If Q meets conditional stability (simple eigenvalue of 1, all others within unit disk) then p* is guaranteed to converge to $p^* = y^* - \tau y^* \mathbf{1}_n$ with τ left eigenvector of G: $\tau \mathbf{1}_n = 1$

$n(M+1)$ eigenvalues of matrix Q found from roots of characteristic polynomials:

$$P_k(s) = s^{M+1} - \sum_{j=0}^{M} \left(a_j + \lambda_k b_j\right) s^{M-j} = 0 \ \text{ for } k = 1, 2, ..., n$$

with roots s_{kj} such that $s_{11} = 1$, and otherwise $\left|s_{kj}\right| < 1$ when conditionally stable

Now define generalized convergence rate as:

$$\varepsilon = 1 - \max_{s_{kj} \neq s_{11} = 1} |s_{kj}| \tag{3}$$

which has exact expressions for low order (M=0 or 1) controllers but becomes intractable for higher orders (however, it is conjectured that higher order controllers would have limited additional benefit). For instance, for M=0

$$\varepsilon = 1 - \max_{k=2,\ldots,n} |1 - b_0 + \lambda_k b_0| \tag{4}$$

The maximal convergence ε* can be found over all strongly connected graphs by solving a quadratic optimization program. It was also proven that only the complete graph has the fastest convergence, ε=1.

Stability Margin

To obtain a measure of stability margin without resorting to gain and phase margins, we first obtain a closed loop transfer function by taking Z-transform as:

$$P(z) = \left(\sum_{j=0}^{M} b_j z^{-j}\right)\left(\left(z - \sum_{j=0}^{M} a_j z^{-j}\right) I - \left(\sum_{j=0}^{M} b_j z^{-j}\right) G\right)^{-1} LY(z)$$

$$P(z) = F(z)Y(z) \text{ where } P(z) = \sum_{k=0}^{\infty} p_k z^{-k}, \text{ and similar for } Q(z), Y(z)$$

The open loop (forward path) transfer function is then:

$$F(z) = \left(I + \Phi(z)\right)^{-1} \Phi(z), \quad \text{or} \quad \Phi(z) = F(z)\left(I - F(z)\right)^{-1}$$

Now compute the inverse of the sensitivity function (defined as the magnitude of this vector from -1 to the closest point on the Nyquist curve plotted for the open loop transfer function):

$$S = \left(I + \Phi(z)\right)^{-1}, \quad \text{with } S_{\max} = \max\left|\left(I + \Phi(z)\right)^{-1}\right| \quad \text{so } \delta_m = \min_{|z|=1}\left|I + \Phi(z)\right|$$

From this the stability margin is defined as:

$$\delta = \min_{k=2,\dots,n} \min_{|z|=1} \left|\frac{P_k(z)}{P_1(z)}\right| \qquad (5)$$

Which again is easily computed for low order controllers (M=0,1,2) but not so easily for higher orders. For example, for M=0,

$$\delta = \min\left|\frac{\mathrm{Re}\left[1-\lambda_k\right]}{\left[1-\lambda_k\right]} - \frac{b_0}{2}\left|1-\lambda_k\right|\right|, \qquad a_0 = 1 - b_0 \qquad (6$$

Pareto Framework.

Now we turn to constructing the 2-dimensional manifold or efficient frontier of convergence rate and stability margin. For a graph G, we can formulate the following constrained optimization problem:

$$\delta_G^*(\varepsilon) = \max_{\substack{a_0,\ldots,a_M \\ b_0,\ldots,b_M}} \min_{k=2,\ldots,n} \min_{|z|=1} \left| \frac{P_k(z)}{P_1(z)} \right|$$

$$\begin{aligned} \text{s.t.} \quad & |s_{1i}| \le 1-\varepsilon, \quad P_1(s_{1i}) = 0, \quad i = 2,\ldots,M+1, \quad M \ge 1, \\ & |s_{ki}| \le 1-\varepsilon, \quad P_k(s_{ki}) = 0, \quad i = 1,\ldots,M+1, \quad k = 2,\ldots,n, \quad M \ge 0, \\ & \textstyle\sum_{j=0}^{M}(a_j + b_j) = 1, \end{aligned}$$

Which has a solution that optimizes controller gains a_j and b_j to find the maximal stability margin $\delta^* G(\varepsilon)$ for $\varepsilon \in (0,\varepsilon*]$. This is the efficient frontier.

Results.

An efficient frontier was computed for the example graphs in Figure 1. The results are in Figure 2 and 3.

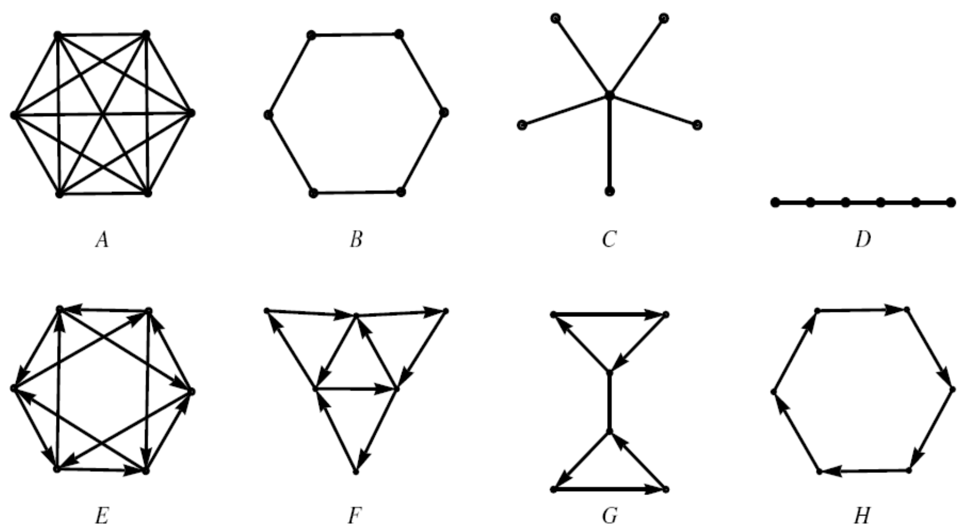

Figure 1: Example Graph Topologies on 6 Nodes.

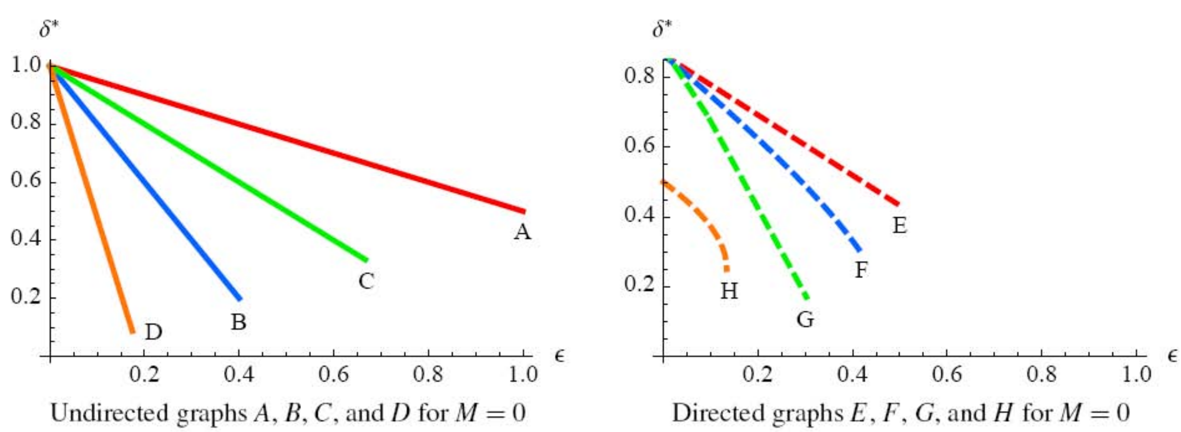

Figure 2. Efficient Frontier For Graphs in Figure 1 with M=0

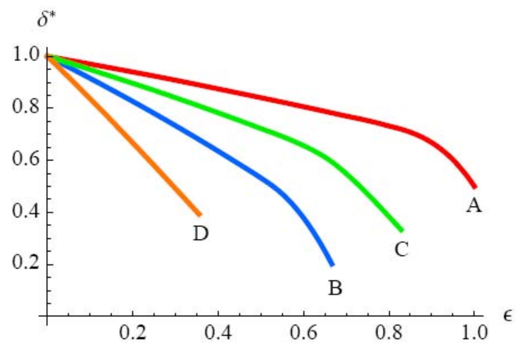

Figure 3. Efficient Frontier For Graphs in Figure 1 with M=1

As can be seen, the complete graph always dominates but other graphs do show dominating effects as well. These results show how stability margin can be traded for convergence rate and vice versa. For the zero-order information loop and all strongly connected communication graphs, efficient frontiers have been found analytically, while for the first-order information loop and undirected communication graphs, they have been evaluated numerically. The result that the complete graph has the highest convergence rate and the best efficient frontier can be related to the fact that the normalized adjacency matrix of the complete graph has the minimal number of distinct eigenvalues. In other words, the more distinct eigenvalues the normalized adjacency matrix of a graph has, the more constraints on control gains in the information loop are imposed, and consequently, the lower maximal convergence rate and maximal stability margin are. The complete graph is the primal choice for vehicle communication. However, if for some reason, it cannot be afforded, numerical results show that the "star" graph is the next best choice.

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Publications

"Optimization Framework for Stability and Convergence of Information Flow in Cooperative Systems," Zabarankin, M., Murphey, R., Murray, R., IEEE Transactions on Automatic Control, accepted 2011.

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